Image Edge Detection Based on Wavelet Multi-scale Transform

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Keywords: image edge; wavelet transform; multi-scale analysis

Abstract: Edge detection is one of the most important techniques in image processing. This paper presents a method of multi-scale edge image reconstruction, which can get the sharp change points in different scales. The theoretical basis of image edge detection based on local extremum is given. Finally, the equivalent condition of edge reconstruction is given by wavelet multiresolution analysis.

1. Introduction

The most basic feature of an image is the edge[1-3]. The edge can be understood as the discontinuity of the local feature of the image, which is represented by the mutation of the gray level of the image, the mutation of the texture structure and the change of the color. For image processing, the edge represents the signal mutation and contains a lot of information in the image, which can be used for image analysis, target recognition and image filtering. Most of the main information of the image exists in the edge of the image, which is mainly manifested by the discontinuity of the local characteristics of the image, and is the place where the gray level changes strongly. In the field of digital image, edge detection is an important image preprocessing technology, which is widely used in contour, feature extraction and texture analysis.

Wavelet transform is widely used in image processing[4-12]. Multi-scale analysis of signal by wavelet transform is very suitable for extracting local features of signal. The classical edge detection method, because of the introduction of various forms of differential operation, will inevitably cause extreme sensitivity to noise. The result of edge detection on it is often to detect the noise as an edge point, and the real edge is not detected because of noise interference. Therefore, for noisy image, a good edge detection method should have a good ability of noise suppression, but also have a complete feature of edge preservation.

Wavelet transform can provide a good denoising method. When the wavelet generating function is the first derivative of the smoothing function, the modulus of the wavelet transform of the signal takes the local maximum at the signal mutation point. The wavelet transform of image on different scales provides certain edge information.When the image is small-scale, the edge detail information is rich, the edge positioning accuracy is high, but it is easy to be disturbed by noise. When the image is large-scale, the edge is stable, anti noise is good, but the positioning accuracy is poor. In practical application, there is often a contradiction between noise removal and accurate positioning.

The basic idea of multi-scale edge detection is to detect the transformation of the maximum value of the edge modulus at the corresponding point by using several edge detection operators of different scales along the gradient direction, and to get the final edge image by selecting the threshold value and then synthesizing at different scales, which can better solve the contradiction between noise and positioning accuracy.

2. Fundamental Theory

Multi scale edge detection is to polish the original signal at different scales, and then detect the sharp change point of the original signal from the first or second derivative of the polished signal.

Let $\theta(x)$ be a smoothing function, which satisfy $\int_{-\infty}^{+\infty}\theta(x)dx = 1$ and $\lim_{x\to\pm\infty}\theta(x) = 0$ .Then $\theta(x)$ can be Gauss function and normal B-spline function and so on. Let $\theta(x)$ be two order derivable, then $\theta(x)$ can be define that $\varphi^{\mathrm{I}}(x)=\frac{d}{dx}\theta(x)$, $\varphi^{\mathrm{II}}(x)=\frac{d^2}{dx^2}\theta(x)$. Then we have $\int_{-\infty}^{+\infty}\varphi^{\mathrm{I}}(x)dx=0$ and $\int_{-\infty}^{+\infty}\varphi^{\mathrm{II}}(x)dx=0$, so $\varphi^{\mathrm{I}}(x)$ and $\varphi^{\mathrm{II}}(x)$ are wavelet.

Notice, the definition of convolution of f,g is $f*g(x)=(f*g)(x):=\int_{-\infty}^{+\infty}f(u)g(x-u)du$. Let $\zeta_s(x)=\frac{1}{s}\zeta(\frac{x}{s})$,then on the scale s and position x of wavelet $\varphi^{\mathrm{I}}(x)$ and $\varphi^{\mathrm{II}}(x)$, the Normalized wavelet transform is defined as

$$W_s^{\mathrm{I}}f(x)=f*\varphi_s^{\mathrm{I}}(x),W_s^{\mathrm{II}}f(x)=f*\varphi_s^{\mathrm{II}}(x). \tag{1}$$

The usual wavelet transform is defined as

$$(W_h f)(a,b)=<f,h_{a,b}>=|a|^{-\frac{1}{2}}\int_{-\infty}^{+\infty}f(t)\overline{h(\frac{t-b}{a})}dt\,,h_{a,b}=|a|^{-\frac{1}{2}}h(\frac{t-b}{a}). \tag{2}$$

(1) and (2) can convert to each other. In fact, let $\varphi(t)=\overline{h(-t)}$,then

$$W_s f(x)=|s|^{-\frac{1}{2}}sign(s)(W_h f)(s,x) \tag{3}$$

Form(1), we obtained that

$$W_s^{\mathrm{I}}f(x)=f*(s\frac{d\theta_s}{dx})(x)=s\frac{d}{dx}(f*\theta_s)(x) \tag{4}$$

$$W_s^{\mathrm{II}}f(x)=f*(s^2\frac{d^2\theta_s}{dx^2})(x)=s^2\frac{d^2}{dx^2}(f*\theta_s)(x) \tag{5}$$

In fact, $f*(s\frac{d\theta_s}{dx})(x)=\frac{1}{s}\int_{-\infty}^{+\infty}f(t)s\frac{d\theta_s}{dx}(\frac{x-t}{s})dt$

$$=\int_{-\infty}^{+\infty}f(t)\frac{d}{dx}\theta(\frac{x-t}{s})dt=\frac{d}{dx}\int_{-\infty}^{+\infty}f(t)\theta(\frac{x-t}{s})dt$$

$$=s\frac{d}{dx}s\int_{-\infty}^{+\infty}f(t)\theta(\frac{x-t}{s})dt=s\frac{d}{dx}(f*\theta_s)(x)$$

In the same way, (2) can be proved.

The normalized wavelet transform of $f(x)$ about $\varphi^{\mathrm{I}}(x)$ and $\varphi^{\mathrm{II}}(x)$ are the product of the first derivatives $f*g(x)$ of s and s, the product of the second derivatives $f*g(x)$ of s and s^2. The local extremum of $W_s^{\mathrm{I}}f(x)$ Correspond to the zero crossing point of $W_s^{\mathrm{II}}f(x)$ and the inflection point of $f*\theta_s(x)$. Specially, let $\theta(x)$ be Gauss function , then the zero cross detection is edge detection of Marr-Hildreth and extrema detection is edge detection of Canny.

Because the large scale of s, the convolution of signal and $\theta_s(x)$ can eliminate the small changes in the signal, so only large sharp changes can be detected, which is just the detection of low-frequency signal in wavelet decomposition. Therefore, for different the values of s, we can get the sharp change points under different scales, which is multi-scale edge detection, equivalent to the detection of signals in different frequency bands after wavelet decomposition.

Detection of zero crossing or local extremum is a similar method, but the method of finding local extremum has its advantages. The inflection point of $f*\theta_s(x)$ is the maximum or minimum value of its first derivative. It is difficult to distinguish these two types of zero crossing points by second derivative. However, with the first derivative, it is easy to find the sharp change point and the value of $|W_s^{\mathrm{I}} f(x)|$ at this point by detecting the local maximum of $W_s^{\mathrm{I}} f(x)$.

The edge detection is extended to two dimensions, and the local maximum becomes the gradient vector modulus maximum. Let $\theta(x,y)$ be smoothing function, then we define

$$\varphi^1(x,y)=\frac{\partial\theta(x,y)}{\partial x},\varphi^2(x,y)=\frac{\partial\theta(x,y)}{\partial y} \quad . \tag{6}$$

Then the function $\varphi^{\mathrm{I}}(x,y)$ and $\varphi^{\mathrm{II}}(x,y)$ are two dimensional wavelet. Let $s=2^j$, we note

$$\varphi^1_{2^j}(x,y)=\frac{1}{2^{2j}}\varphi^1(\frac{x}{2^j},\frac{y}{2^j}),\varphi^2_{2^j}(x,y)=\frac{1}{2^{2j}}\varphi^2(\frac{x}{2^j},\frac{y}{2^j}). \tag{7}$$

then the normalized wavelet transform of $f(x,y)\in L^2(R^2)$ on $\varphi^1(x,y)$ and $\varphi^2(x,y)$ have two components

$$W^1_{2^j} f(x,y)=f*\varphi^1_{2^j}(x,y),W^2_{2^j} f(x,y)=f*\varphi^2_{2^j}(x,y). \tag{8}$$

Similar to one dimension, it is easy to be prove that

$$\begin{bmatrix} W^1_{2^j} f(x,y) \\ W^2_{2^j} f(x,y)\end{bmatrix}=2^j\begin{bmatrix} \frac{\partial}{\partial x}(f*\theta^j_2(x,y)) \\ \frac{\partial}{\partial y}(f*\theta^j_2(x,y))\end{bmatrix}=2^j\vec{\nabla}(f*\theta^j_2)(x,y). \tag{9}$$

At this time, the right end is 2^j times of the gradient of smooth function. In the actual calculation, $\theta(x,y)$ often takes the form of product of x and y.

3. Image Reconstruction of Multiscale Edge

Supposed that $f(x,y)\in L^2(R^2)$, $(W^1_{2^j} f(x,y),W^2_{2^j} f(x,y))_{j\in Z}$ is the binary wavelet of (8). $\hat{\varphi}^1(\omega_x,\omega_y)$ and $\hat{\varphi}^2(\omega_x,\omega_y)$ are the Fourier transforms of $\varphi^1(x,y)$ and $\varphi^2(x,y)$, respectively. Then the Fourier transforms of $W^1_{2^j} f(x,y)$ and $W^2_{2^j} f(x,y)$ are

$$\begin{cases}\hat{W}^1_{2^j} f(\omega_x,\omega_y)=\hat{f}(\omega_x,\omega_y)\hat{\varphi}^1(2^j\omega_x,2^j\omega_y) \\ \hat{W}^2_{2^j} f(\omega_x,\omega_y)=\hat{f}(\omega_x,\omega_y)\hat{\varphi}^2(2^j\omega_x,2^j\omega_y)\end{cases}. \tag{10}$$

If the local maximum point of dyadic wavelet transform of $f(x,y)$ is (x^j_v,y^j_v). At this point, $M_{2^j} f(x^j_v,y^j_v)=\sqrt{|W^1_{2^j} f(x^j_v,y^j_v)|^2+|W^2_{2^j} f(x^j_v,y^j_v)|^2}$ is local maximum along the gradient direction given by $A_{2^j} f(x^j_v,y^j_v)=\arctan\frac{|W^1_{2^j} f(x^j_v,y^j_v)|}{|W^2_{2^j} f(x^j_v,y^j_v)|}$. Then we obtained $W^1_{2^j} f(x^j_v,y^j_v)$ and $W^2_{2^j} f(x^j_v,y^j_v)$ by $M_{2^j} f(x^j_v,y^j_v)$ and $A_{2^j} f(x^j_v,y^j_v)$. So we can reconstruct the image $f(x,y)$. This problem is equivalent to finding function $h(x,y)$ satisfying the following conditions.

(1) For each scale 2^j and each modulus maximum point (x^j_v,y^j_v), we have

$$W^1_{2^j} h(x^j_v,y^j_v)=W^1_{2^j} f(x^j_v,y^j_v),W^2_{2^j} h(x^j_v,y^j_v)=W^2_{2^j} f(x^j_v,y^j_v). \tag{11}$$

(2) For each scale 2^j, It is found that the position of modulus maximum point is $(x^j_v,y^j_v)_{v\in R}$ by $W^1_{2^j} h(x,y)$ and $W^2_{2^j} h(x,y)$.

For any point (x_0,y_0), wavelet transform can be written as

$$W_{2^j}^1 h(x_0, y_0) = < f(x,y), \varphi_{2^j}^1 (x_0 - x, y_0 - y) >,$$

$$W_{2^j}^2 h(x_0, y_0) = < f(x,y), \varphi_{2^j}^2 (x_0 - x, y_0 - y) >. \quad (12)$$

Let U be the space formed by function family $\{2^j \varphi_{2^j}^1 (x_v^j - x, y_v^j - y), 2^j \varphi_{2^j}^2 (x_v^j - x, y_v^j - y)\}_{(j,v)\in Z\times R}$.

Let O be the orthogonal complement of U in $L^2(R^2)$, then we have $h(x,y) = f(x,y) + g(x,y)$, $g(x,y)\in O$ .We define $\| h \|_*^2 = \sum_{j=-\infty}^{+\infty} [\| W_{2^j}^1 h \|^2 + \| W_{2^j}^2 h \|^2 + 2^{2j} (\| \frac{\partial W_{2^j}^1 h}{\partial x} \|^2 + \| \frac{\partial W_{2^j}^2 h}{\partial y} \|^2)$, the minimization of this norm produces a wavelet transform with horizontal and vertical components, and the norm of $L^2(R^2)$ is as small as possible, which is related to (1), so that the local maximum can be generated at point (x_v^j, y_v^j).

4. Conclusion

Multi-scale detection method is mainly through the effective combination of multiple edge detection operators of different scales to correctly detect the edge of the image. The method of finding local extremum is used to detect image edge. Image reconstruction based on modulus maxima information of different scales of image in wavelet transform. The theoretical basis of image edge detection based on local extremum is given. The equivalent condition of edge reconstruction is given by wavelet multiresolution analysis.

Acknowledgements

This research was financially supported by the Special Scientific Research Project of Shaanxi Provincial Department of Education (Grant NO. 19JK0635) .

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